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Application transmittal

UTILITY  
PATENT APPLICATION  
TRANSMITTAL

Only for new non-provisional applications under 37 CFR 1.53(b))

Attorney Docket No. **AI-Dhahir 1**First Named Inventor or Application Identifier **Naofal Al-Dhahir**Title **Finite-Length Equalization Over Multi-Input Multi-Output Channels**Express Mail Label no. **EK508640581US**To: **Assistant Commissioner for Patents  
Box Patent Application  
Washington D.C. 20231**

## APPLICATION ELEMENTS

- ☒ Fee Transmittal Form (original and duplicate)
- ☒ Specification Total Pages **18**  
title  
cross reference to related applications (e.g. provisional application)  
background  
summary  
brief description of the drawings (if filed)  
detailed description  
claims  
abstract
- ☒ Drawing(s) Total Pages **4**
- ☒ Declaration Total Pages **2**  
a. ☐ Newly executed  
b. ☐ Copy from a prior application (37 CFR 1.63(d))  
(for continuations/divisionals with section below filled out)  
☐ Deletion of Inventor(s) Signed Statement attached deleting  
inventor(s) named in the prior application. 37 CFR 163 (d)(2)  
and 1.33(b).
- Incorporation by reference (usable if Declaration is a copy):  
The entire disclosure of the prior application, from which a copy of the oath or declaration  
is supplied, is considered as being part of the disclosure of the accompanying application  
is hereby incorporated by reference herein.
- Other

## ACCOMPANYING APPLICATION PARTS

- ☒ Assignment
- ☒ Recordation form
- ☒ Power of Attorney
- ☒ Postcard
- ☐ Small entity statement
- ☐ Certified copy of priority documents
- ☒ Information disclosure statement
- ☒ Copies of IDS citations
- ☐ 37 CFR 3.73(b) Statement
- ☒ check
- ☐ Other

If a CONTINUING APPLICATION, check appropriate box and supply the requisite information:

☒ Continuation ☐ Divisional ☐ Continuation-in-part (CIP) of prior Application No:

## CORRESPONDENCE ADDRESS

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Date


9/27/00

I hereby certify that this Application is being deposited with the United States Postal Service "Express Mail Post Office to Addressee" service under 37 CFR 1.10 on the date indicated above and is addressed to the Assistant Commissioner for Patents, Washington D.C. 20231.

9/28/00  
Date of Deposit**Henry Brendzel**  
(Printed Name of Person Mailing Paper)**Henry Brendzel**  
(Signature of Person Mailing Paper)

<b>FEE TRANSMITTAL</b> <i>Patent Fees are subject to annual revisions on October 1.          These are the fees effective November 10, 1998          Small entity payments must be supported by a small entity statement,          Otherwise, large entity fees must be paid. See Forms PTO/SB/09-12.</i>		<b>Complete if Known</b>	
		Application Number	60/158714
		Filing Date	10/8/99
		First Named Inventor	Naofal Al-Dhahir
		Examiner Name	
TOTAL AMOUNT OF PAYMENT		(\$)	730
		Attorney Docket ID	Al-Dhahir 1

<b>METHOD OF PAYMENT</b> (check one)		<b>FEE CALCULATION</b> (continued)																																																															
1. The Commissioner is hereby authorized to charge indicated fees and other underpayments, and credit overpayments to:  Deposit Account Number      Deposit Account Name <b>500732</b> <b>Henry T. Brendzel</b>  <input checked="" type="checkbox"/> Charge any additional Fee Required under 37 CFR 1.16 and 1.17 <input type="checkbox"/> Charge the Issue Fee Set in 37 CFR 1.18 at the Mailing of the Notice of Allowance  2. <input type="checkbox"/> Payment enclosed: <input type="checkbox"/> Check <input type="checkbox"/> Money Order <input type="checkbox"/> Other		<b>3. ADDITIONAL FEES</b>																																																															
<b>FEE CALCULATION</b>		<table border="1"> <thead> <tr> <th>Fee Description</th> <th>Fee Paid</th> </tr> </thead> <tbody> <tr><td>Surcharge - late filing fee or oath</td><td></td></tr> <tr><td>Surcharge - late provisional filing fee or cover sheet</td><td></td></tr> <tr><td>Non-English specification</td><td></td></tr> <tr><td>For filing request for reexamination</td><td></td></tr> <tr><td>Requesting publication of SIR prior to Examiner action</td><td></td></tr> <tr><td>Requesting publication of SIR after Examiner action</td><td></td></tr> <tr><td>Extension for reply within the first month</td><td></td></tr> <tr><td>Extension for reply within the second month</td><td></td></tr> <tr><td>Extension for reply within the third month</td><td></td></tr> <tr><td>Extension for reply within the fourth month</td><td></td></tr> <tr><td>Extension for reply within the fifth month</td><td></td></tr> <tr><td>Notice of Appeal</td><td></td></tr> <tr><td>Filing a brief in support of an appeal</td><td></td></tr> <tr><td>Requesting an oral hearing</td><td></td></tr> <tr><td>Petition to institute a public use proceeding</td><td></td></tr> <tr><td>Petition to revive - unavoidable</td><td></td></tr> <tr><td>Petition to revive - unintentional</td><td></td></tr> <tr><td>Utility issue fee (or reissue)</td><td></td></tr> <tr><td>Design issue fee</td><td></td></tr> <tr><td>Plant issue fee</td><td></td></tr> <tr><td>Petitions to the Commissioner</td><td></td></tr> <tr><td>Petitions related to provisional applications</td><td></td></tr> <tr><td>Submission of Information Disclosure Statement</td><td></td></tr> <tr><td>Recording each patent assignment per property (times number of properties)</td><td></td></tr> <tr><td>Filing a submission after final rejection (37 CFR 1.129(a))</td><td></td></tr> <tr><td>For each additional invention to be examined (37 CFR 1.129(b))</td><td></td></tr> <tr><td>Other fee</td><td></td></tr> <tr><td>Other fee</td><td></td></tr> <tr><td>Other fee</td><td></td></tr> <tr> <td><b>SUBTOTAL (2) (\$)</b></td> <td><b>40</b></td> </tr> </tbody> </table>		Fee Description	Fee Paid	Surcharge - late filing fee or oath		Surcharge - late provisional filing fee or cover sheet		Non-English specification		For filing request for reexamination		Requesting publication of SIR prior to Examiner action		Requesting publication of SIR after Examiner action		Extension for reply within the first month		Extension for reply within the second month		Extension for reply within the third month		Extension for reply within the fourth month		Extension for reply within the fifth month		Notice of Appeal		Filing a brief in support of an appeal		Requesting an oral hearing		Petition to institute a public use proceeding		Petition to revive - unavoidable		Petition to revive - unintentional		Utility issue fee (or reissue)		Design issue fee		Plant issue fee		Petitions to the Commissioner		Petitions related to provisional applications		Submission of Information Disclosure Statement		Recording each patent assignment per property (times number of properties)		Filing a submission after final rejection (37 CFR 1.129(a))		For each additional invention to be examined (37 CFR 1.129(b))		Other fee		Other fee		Other fee		<b>SUBTOTAL (2) (\$)</b>	<b>40</b>
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<b>SUBMITTED BY</b>				<b>Complete (if applicable)</b>	
Typed of Printed Name	Henry T. Brendzel			Reg. Number	26,844
Signature				Date	9/27/00
				Deposit Account User ID	

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**Application Information**

20 Title Line One::

Finite-Length Equalization OverMulti-  
Input Multi-Output Channels

Total Drawing Sheets::

Formal Drawings?::

Yes

Application Type::

25

**Continuity Information**

This application is a::

converted utility application of Provisional  
60/158,714

> Application One::

Filing Date::

October 8, 1999

30

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## Finite-Length Equalization Over Multi-Input Multi-Output Channels

### Related Application:

5           This application claims priority from Provisional application No. 60/158,714, filed on October 8, 1999. This application is also related to a Provisional application No. 60/158,713, also filed on October 8, 1999.

### Background of the Invention:

10           In multi-user communication over linear, dispersive, and noisy channels, the received signal is composed of the sum of several transmitted signals corrupted by inter-symbol interference, inter-user interference, and noise. Examples include TDMA digital cellular systems with multiple transmit/receive antennas, wide-band asynchronous CDMA systems, where inter-user interference is also known as  
15           multiple access interference, wide-band transmission over digital subscriber lines (DSL) where inter-user interference takes the form of near-end and far-end crosstalk between adjacent twisted pairs, and in high-density digital magnetic recording where inter-user interference is due to interference from adjacent tracks.

20           Multi-user detection techniques for multi-input multi-output (MIMO) systems have been shown to offer significant performance advantages over single user detection techniques that treat inter-user interference as additive colored noise and lumps its effects with thermal noise. Recently, it has been shown that the presence of inter-symbol interference in these MIMO systems could enhance overall system capacity, provided that effective multi-user detection techniques are  
25           employed.

30           The optimum maximum likelihood sequence estimation (MLSE) receiver for MIMO channels was developed by S. Verdu, "Minimum Probability of Error for Asynchronous Gaussian Multiple Access Channels," *IEEE Transactions on Information Theory*, January 1986, pp. 85-96. However, its exponential complexity increases with the number of users, and channel memory makes its implementation costly for multi-user detection on severe-inter-symbol interference channels.

Two alternative transceiver structures have been recently proposed for MIMO dispersive channels as well. These structures, which are widely used in practice for single-input single-output dispersive channels, are the Discrete Multitone and minimum-mean-square-error decision feedback equalizer (MMSE-DFE). In the latter category, this includes A. Duel-Hallen "Equalizers for Multiple Input Multiple Output Channels and PAN Systems with Cyclostationary Input Sequences," *IEEE Journal on Selected Areas on Communications*, April 1992, pp. 630-639; A. Duel-Hallen "A Family of Multiuser Decision-Feedback Detectors for Asynchronous Code Division Multiple Access Channels," *IEEE Transactions on Communications*, Feb/Mar/April 1995, pp. 421-434; J. Yang et al "Joint Transmitter and Receiver Optimization for Multiple Input Multiple Output Systems with Decision Feedback," *IEEE Transactions on Information Theory*, Sep. 1994, pp. 1334-1347; and J. Yang et al "On Joint Transmitter and Receiver Optimization for Multiple Input Multiple Output (MIMO) Transmission Systems," *IEEE Transactions on Communications*, Dec. 1994, pp. 3221-3231. Alas, the prior art does not offer a practical MIMO MMSE-DFE receiver with feedforward and feedback FIR filters whose coefficients can be computed in a single computation (i.e., non-iteratively) in real-time under various MIMO detection scenarios.

## **Summary**

An advance in the art is realized with receiver having a multiple number of receiving antennas that feed a MIMO feedforward filter that is constructed from FIR filters with coefficients that are computed based on environment parameters that are designer-specified. Signals that are derived from a multiple-output feedback filter structure are subtracted from the signals from the multiple outputs of the feedforward filter structure, and the resulting difference signals are applied to a decision circuit. Given a transmission channel that is modeled as a set of FIR filters with memory  $\nu$ , a matrix  $\mathbf{W}$  is computed for a feedforward filter that results in an effective transmission channel  $\mathbf{B}$  with memory  $N_b$ , where  $N_b \leq \nu$ , where  $\mathbf{B}$  is optimized so that  $\mathbf{B}_{opt} = \operatorname{argmin}_{\mathbf{B}} \operatorname{trace}(\mathbf{R}_{ee})$  subject to selected constraints;  $\mathbf{R}_{ee}$  being the error autocorrelation function. The feedback filter is modeled by

$\begin{bmatrix} \mathbf{I}_{n_i} & \mathbf{0}_{n_i \times n_i N_b} \end{bmatrix} - \mathbf{B}^*$ , where  $n_i$  is the number of outputs in the feedforward filter, as well as the number of outputs in the feedback filter.

The coefficients of feedforward and the feedback filters, which are sensitive to a variety of constraints that can be specified by the designer, are computed by a processor in a non-iterative manner, only as often as it is expected for the channel characteristics to change.

### **Brief Description of the Drawing**

FIG. 1 shows the major elements of a receiver in accord with the principles disclosed herein;

FIG. 2 presents the structure of elements 23 and 26;

FIG. 3 is a flowchart describing one method carried out by processor 22;

and

FIG. 4 is a flowchart describing another method carried out by processor 22.

### **Detailed Description**

FIG. 1 depicts the general case of an arrangement with  $n_t$  transmitting antennas 11-1, 11-2, ... 11- $n_t$ , that output signals (e.g., space-time encoded signals) to a transmission channel, and  $n_o$  receiving antennas 21-1, 21-2, ... 21- $n_o$ . Each transmitting antenna  $p$  outputs a complex-valued signal  $x_p$ , the signals of the  $n_t$  antennas pass through a noisy transmission channel, and the  $n_o$  receiving antennas capture the signals that passed through the transmission channel. The received signals are oversampled by a factor of  $L$  in element 20 and applied to feedforward  $\mathbf{W}$  filters 23. Thus, the sampling clock at the output of element 20 is of period  $T_s = T/L$ , where  $T$  is the inter-symbol period at the transmitting antennas. The transmission channel's characterization is also referenced to  $T_s$ .

Filter bank 23 delivers an  $n_i$  plurality of output signals ( $n_i$  can equal  $n_t$  for example) from which feedback signals are subtracted in circuit 24 and applied to decision circuits 25 (comprising conventional slicers). The outputs of decision circuits 25 are applied to feedback filters 26, which develop the feedback signals.

Processor 22 develops the filter coefficients for the filters within elements 23 and 26 and installs the coefficients in the filters within these elements, as disclosed in detail below.

In the illustrative embodiment disclosed herein, the received signal is expressed by

$$y_k^{(j)} = \sum_{i=1}^N \sum_{m=0}^{v^{(i,j)}} h_m^{(i,j)} x_{k-m}^{(i)} + n_k^{(j)}, \quad (1)$$

where  $y_k^{(j)}$  is the signal at time  $k$  at the  $j^{\text{th}}$  receiving antenna,  $h_m^{(i,j)}$  is the  $m^{\text{th}}$  coefficient (tap) in the channel impulse response between the  $i^{\text{th}}$  transmitting antenna and the  $j^{\text{th}}$  receiving antenna, and  $n_k^{(j)}$  is the noise vector at the  $j^{\text{th}}$  receiving antenna. The memory of this path (i.e., the largest value of  $m$  for which  $h_m^{(i,j)}$  is not zero) is denoted by  $v^{(i,j)}$ .

It may be noted that it not unreasonable to assume, that the memory of the transmission channel is the same for all  $i,j$  links ( $n_i \times n_o$  such links), in which case  $v^{(i,j)} = v$ . Alternatively, the  $v^{(i,j)}$  limit in equation (1) can be set to that  $v$  which corresponds to maximum length of all of the  $n_i \times n_o$  channel input responses, i.e.,  $v = \max_{i,j} v^{(i,j)}$ . It may also be noted that all of the variables in equation (1) are actually  $l \times 1$  column vectors, corresponding to the  $l$  time samples per symbol in the oversampled FIG. 1 arrangement.

By grouping the received samples from all  $n_o$  antennas at symbol time  $k$  into an  $n_o l \times 1$  column vector  $\mathbf{y}_k$ , one can relate  $\mathbf{y}_k$  to the corresponding  $n_i \times 1$  (column) vector of input samples as follows

$$\mathbf{y}_k = \sum_{m=0}^v \mathbf{H}_m \mathbf{x}_{k-m} + \mathbf{n}_k, \quad (2)$$

where  $\mathbf{H}_m$  is the MIMO channel coefficients matrix of size  $n_o l \times n_i$ ,  $\mathbf{x}_{k-m}$  is a size  $n_i \times 1$  input (column) vector, and  $\mathbf{n}_k$  is a size  $n_o l \times 1$  vector.

Over a block of  $N_f$  symbol periods, equation (2) can be expressed in matrix notation as follows:

$$\begin{bmatrix} \mathbf{y}_{k+N_f-1} \\ \mathbf{y}_{k+N_f-2} \\ \vdots \\ \mathbf{y}_k \end{bmatrix} = \begin{bmatrix} \mathbf{H}_0 & \mathbf{H}_1 & \cdots & \mathbf{H}_v & 0 & \cdots & 0 \\ 0 & \mathbf{H}_0 & \mathbf{H}_1 & \cdots & \mathbf{H}_v & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \mathbf{H}_0 & \mathbf{H}_1 & \cdots & \mathbf{H}_v \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k+N_f-1} \\ \mathbf{x}_{k+N_f-2} \\ \vdots \\ \mathbf{x}_{k-v} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_{k+N_f-1} \\ \mathbf{n}_{k+N_f-2} \\ \vdots \\ \mathbf{n}_k \end{bmatrix} \quad (3)$$

or, more compactly,

$$\mathbf{y}_{k+N_f-1:k} = \mathbf{H} \mathbf{x}_{k+N_f-1:k-v} + \mathbf{n}_{k+N_f-1:k} \quad (4)$$

The subscripts in equation (4) indicate a range. For example  $k + N_f - 1 : k$  means the range from  $k + N_f - 1$  to  $k$ , inclusively.

It is useful to define the following correlation matrices:

$$\mathbf{R}_{xy} \equiv E[\mathbf{x}_{k+N_f-1:k-v} \mathbf{y}_{k+N_f-1:k}^*] = \mathbf{R}_{xx} \mathbf{H}^* \quad (5)$$

$$\mathbf{R}_{yy} \equiv E[\mathbf{y}_{k+N_f-1:k} \mathbf{y}_{k+N_f-1:k}^*] = \mathbf{H} \mathbf{R}_{xx} \mathbf{H}^* + \mathbf{R}_{nn} \quad (6)$$

$$\mathbf{R}_{xx} \equiv E[\mathbf{x}_{k+N_f-1:k-v} \mathbf{x}_{k+N_f-1:k-v}^*] \text{ and} \quad (7)$$

$$\mathbf{R}_{nn} \equiv E[\mathbf{n}_{k+N_f-1:k} \mathbf{n}_{k+N_f-1:k}^*], \quad (8)$$

and it is assumed that these correlation matrices do not change significantly in time or, at least, do not change significantly over a time interval that corresponds to a TDMA burst (assumed to be much shorter than the channel coherence time), which is much longer than the length of the FIR filters in element 23 (in symbol periods denoted by  $N_f$ ). Accordingly, a re-computation within processor 22 of the above matrices, and the other parameters disclosed herein, leading to the computation of the various filter coefficients, need not take place more often than once every TDMA burst. Once  $\mathbf{H}$ ,  $\mathbf{R}_{xx}$  and  $\mathbf{R}_{nn}$  are ascertained (through the conventional use of training sequences),  $\mathbf{R}_{xy}$  and  $\mathbf{R}_{yy}$  are computed by  $\mathbf{R}_{xx} \mathbf{H}^*$  and  $\mathbf{H} \mathbf{R}_{xx} \mathbf{H}^* + \mathbf{R}_{nn}$ , respectively.

In accordance with the principles disclosed herein, element 23 comprises a collection of FIR filters that are interconnected as shown in FIG. 2, and the impulse response coefficients of element 23 can be expressed by  $\mathbf{W}^* \equiv [\mathbf{W}_0^* \mathbf{W}_1^* \cdots \mathbf{W}_{N_f-1}^*]$ , each having  $N_f$  matrix taps  $\mathbf{W}_i$ , of size  $(l_0 n \times n_i)$ . That is,  $\mathbf{W}_i$  has the form:



$$\mathbf{W}_i = \begin{bmatrix} \mathbf{w}_i^{(1,1)} & \dots & \mathbf{w}_i^{(1,n_i)} \\ \vdots & \dots & \vdots \\ \mathbf{w}_i^{(n_o,1)} & \dots & \mathbf{w}_i^{(n_o,n_i)} \end{bmatrix} \quad (9)$$

where each entry in  $\mathbf{W}_i^{(p,q)}$  is an  $l \times 1$  vector corresponding to the  $l$  output samples per symbol. Stated in other words, the matrix  $\mathbf{W}_0$  specifies the 0<sup>th</sup> tap of the set of filters within element 23, the matrix  $\mathbf{W}_1$  specifies the 1<sup>st</sup> tap of the set of filters within element 23, etc.

Also in accordance with the principles disclosed herein, element 26 comprises a collection of FIR filters that also are interconnected as shown in FIG. 2, and the impulse response coefficients of element 26 is chosen to be equal to

$$\begin{bmatrix} \mathbf{I}_{n_i} & \mathbf{0}_{n_i \times n_i N_b} \end{bmatrix} - \mathbf{B}^* \equiv \begin{bmatrix} (\mathbf{I}_{n_i} - \mathbf{B}_0^*) & \mathbf{B}_1^* & \dots & \mathbf{B}_{N_b}^* \end{bmatrix}, \quad (10)$$

where  $\mathbf{B}^*$  is expressed by  $\mathbf{B}^* \equiv [\mathbf{B}_0^* \mathbf{B}_1^* \dots \mathbf{B}_{N_b}^*]$ , with  $(N_b + 1)$  matrix taps  $\mathbf{B}_i$ , each of size  $n_i \times n_i$ . That is,  $\mathbf{B}_i$  has the form:

$$\mathbf{B}_i = \begin{bmatrix} b_i^{(1,1)} & \dots & b_i^{(1,n_i)} \\ \vdots & \dots & \vdots \\ b_i^{(n_i,1)} & \dots & b_i^{(n_i,n_i)} \end{bmatrix}. \quad (11)$$

Stated in other words,  $\mathbf{B}_0$  specifies the 0<sup>th</sup> tap of the set of filters within element 26, the matrix  $\mathbf{B}_1$  specifies the 1<sup>st</sup> tap of the set of filters within element 26, etc.

Defining  $\tilde{\mathbf{B}}^* \equiv [\mathbf{0}_{n_i \times n_i \Delta_b} \mathbf{B}^*]$ , where  $\tilde{\mathbf{B}}^*$  is a matrix of size  $n_i \times n_i (N_f + v)$ , the value of  $N_b$  is related to the decision delay by the equality  $(\Delta + N_b + 1) = (N_f + v)$ .

The error vector at time  $k$  is given by

$$\mathbf{E}_k = \tilde{\mathbf{B}}^* \mathbf{x}_{k+N_f-\Delta-k-v} - \mathbf{W}^* \mathbf{y}_{k+N_f-\Delta-k}. \quad (12)$$

Therefore, the  $n_i \times n_i$  error auto-correlation matrix is

$$\begin{aligned}
\mathbf{R}_{ee} &\equiv E[\mathbf{E}_k^* \mathbf{E}_k] \\
&= \tilde{\mathbf{B}}^* \mathbf{R}_{xx} \tilde{\mathbf{B}} - \tilde{\mathbf{B}}^* \mathbf{R}_{xy} \mathbf{W} - \mathbf{W}^* \mathbf{R}_{yx} \tilde{\mathbf{B}} + \mathbf{W}^* \mathbf{R}_{yy} \mathbf{W} \\
&= \tilde{\mathbf{B}}^* (\mathbf{R}_{xx} - \mathbf{R}_{xy} \mathbf{R}_{yy}^{-1} \mathbf{R}_{yx}) \tilde{\mathbf{B}} + (\mathbf{W}^* - \tilde{\mathbf{B}}^* \mathbf{R}_{xy} \mathbf{R}_{yy}^{-1}) \mathbf{R}_{yy} (\mathbf{W}^* - \tilde{\mathbf{B}}^* \mathbf{R}_{xy} \mathbf{R}_{yy}^{-1}) \\
&= \tilde{\mathbf{B}}^* \mathbf{R}^\perp \tilde{\mathbf{B}} + \mathbf{G}^* \mathbf{R}_{yy} \mathbf{G}
\end{aligned} \tag{13}$$

Using the *Orthogonality Principle*, which states that  $E[\mathbf{E}_k \mathbf{y}_{k+N_f-1:k}^*] = 0$ , it can be shown that the optimum matrix feedforward and feedback filters are related by

$$\begin{aligned}
\mathbf{W}_{opt}^* &= \tilde{\mathbf{B}}_{opt}^* \mathbf{R}_{xy} \mathbf{R}_{yy}^{-1} \\
&= \tilde{\mathbf{B}}_{opt}^* \mathbf{R}_{xx} \mathbf{H}^* (\mathbf{H} \mathbf{R}_{xx} \mathbf{H}^* + \mathbf{R}_{nn})^{-1} \\
&= \tilde{\mathbf{B}}_{opt}^* (\mathbf{R}_{xx}^{-1} + \mathbf{H}^* \mathbf{R}_{nn}^{-1} \mathbf{H})^{-1} \mathbf{H}^* \mathbf{R}_{nn}^{-1},
\end{aligned} \tag{14}$$

5 and the  $n_i \times n_i$  auto-correlation matrix  $\mathbf{R}_{ee}$  is

$$\begin{aligned}
\mathbf{R}_{ee} &\equiv E[\mathbf{E}_k^* \mathbf{E}_k] \\
&= \tilde{\mathbf{B}}^* (\mathbf{R}_{xx} - \mathbf{R}_{xy} \mathbf{R}_{yy}^{-1} \mathbf{R}_{yx}) \tilde{\mathbf{B}} \\
&= \tilde{\mathbf{B}}^* \mathbf{R}^\perp \tilde{\mathbf{B}} \\
&= \tilde{\mathbf{B}}^* (\mathbf{R}_{xx} - \mathbf{R}_{xx} \mathbf{H}^* (\mathbf{H} \mathbf{R}_{xx} \mathbf{H}^* + \mathbf{R}_{nn})^{-1} \mathbf{H} \mathbf{R}_{xx}) \tilde{\mathbf{B}} \\
&= \tilde{\mathbf{B}}^* (\mathbf{R}_{xx}^{-1} + \mathbf{H}^* \mathbf{R}_{nn}^{-1} \mathbf{H})^{-1} \tilde{\mathbf{B}}.
\end{aligned} \tag{15}$$

$\mathbf{R}_{ee}$  can also be expressed as  $\mathbf{R}_{ee} = \tilde{\mathbf{B}}^* \mathbf{R}^{-1} \tilde{\mathbf{B}}$ , where  $\mathbf{R} = \mathbf{R}_{xx}^{-1} + \mathbf{H}^* \mathbf{R}_{nn}^{-1} \mathbf{H}$ .

It remains to optimize values for the  $\mathbf{B}$  matrix and the  $\mathbf{W}$  matrix such that, in response to specified conditions, the *trace* (or determinant) of  $\mathbf{R}_{ee}$  is minimized.

10 The following discloses three approaches to such optimization.

### Scenario 1

In this scenario, it is chosen to process only previous receiver decisions.

These decisions relate to different users that concurrently have transmitted information that has been captured by antennas 21 and detected in circuit 25.

15 That means that feedback element 26 uses only delayed information and that the 0<sup>th</sup> order coefficients of the filters within element 26 have the value 0. Therefore, in light of the definition expressed in equation (10), this scenario imposes the constraint of  $\mathbf{B}_0 = \mathbf{I}_{n_i}$ .

20 To determine the optimum matrix feedback filter coefficients under this constraint, the following optimization problem needs to be solved:

$$\min_{\tilde{\mathbf{B}}} \mathbf{R}_{ee} = \min_{\tilde{\mathbf{B}}} \tilde{\mathbf{B}}^* \mathbf{R}^{-1} \tilde{\mathbf{B}}, \text{ subject to } \tilde{\mathbf{B}}^* \Phi = \mathbf{C}^*, \quad (16)$$

where

$$\Phi \equiv \begin{bmatrix} \mathbf{I}_{n_i} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{n_i} & \vdots & \vdots \\ \vdots & \mathbf{0} & \ddots & \mathbf{I}_{n_i} \\ \mathbf{0} & \dots & \dots & \mathbf{0} \end{bmatrix} \text{ and } \mathbf{C}^* = \begin{bmatrix} \mathbf{0}_{n_i \times n_i \Delta} & \mathbf{I}_{n_i} \end{bmatrix} \quad (17)$$

It can be shown that the solution to the above is given by

$$\tilde{\mathbf{B}}_{opt} = \mathbf{R} \Phi (\Phi^* \mathbf{R} \Phi)^{-1} \mathbf{C}, \quad (18)$$

resulting in the error signal

$$\mathbf{R}_{ee,min} = \mathbf{C}^* (\Phi^* \mathbf{R} \Phi)^{-1} \mathbf{C}. \quad (19)$$

If we define the partitioning

$$\mathbf{R} \equiv \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{12}^* & \mathbf{R}_{22} \end{bmatrix}, \quad (20)$$

where  $\mathbf{R}_{11}$  is of size  $n_i(\Delta + 1) \times n_i(\Delta + 1)$ , then

$$\tilde{\mathbf{B}}_{opt} = \begin{bmatrix} \mathbf{R}_{11} \\ \mathbf{R}_{12}^* \end{bmatrix} \mathbf{R}_{11}^{-1} \mathbf{C} = \begin{bmatrix} \mathbf{I}_{n_i(\Delta+1)} \\ \mathbf{R}_{12}^* \mathbf{R}_{11}^{-1} \end{bmatrix} \mathbf{C} \quad (21)$$

and

$$\mathbf{R}_{ee,min} = \mathbf{C}^* \mathbf{R}_{11}^{-1} \mathbf{C}, \quad (22)$$

where the delay parameter  $\Delta$  is adjusted to minimize the *trace* (or determinant) of

$\mathbf{R}_{ee,min}$ . Once  $\tilde{\mathbf{B}}_{opt}$  is known, equation (14) is applied to develop  $\mathbf{W}_{opt}^*$ .

FIG. 3 presents a flowchart for carrying out the method of determining the filter coefficients that processor 22 computes pursuant to scenario 1. Step 100 develops an estimate of the MIMO channel between the input points and the output point of the actual transmission channel. This is accomplished in a conventional manner through the use of training sequences. The estimate of the MIMO channel can be chosen to be limited to a given memory length,  $\nu$ , or can be allowed to include as much memory as necessary to reach a selected estimate error level. That, in turn, depends on the environment and is basically equal to the delay spread divided by  $T_s$ .

Following step 100, step 110 determines the matrices,  $\mathbf{R}_{nn}$ ,  $\mathbf{R}_{xx}$ ,  $\mathbf{R}_{xy}$ , and  $\mathbf{R}_{yy}$ . The matrix  $\mathbf{R}_{nn}$  is computed by first computing  $\mathbf{n} = \mathbf{y} - \mathbf{H}\mathbf{x}$  and then computing the expected value  $E[\mathbf{n}^* \mathbf{n}]$  -- see equation (8) above. The matrix  $\mathbf{R}_{xx}$  is computed from the known training sequences -- see equation (7) above -- (or is pre-computed and installed in processor 22). It may be noted that for uncorrelated inputs,  $\mathbf{R}_{xx} = \mathbf{I}$ . The matrices  $\mathbf{R}_{xy}$  and  $\mathbf{R}_{yy}$  are computed from the known training sequences and the received signal or directly from  $\mathbf{H}$  and  $\mathbf{R}_{nn}$  -- see equations (5) and (6) above.

Following step 110, step 120 computes  $\mathbf{R} = \mathbf{R}_{xx}^{-1} - \mathbf{H}^* \mathbf{R}_{nn}^{-1} \mathbf{H}$ , and the partition components,  $\mathbf{R}_{11}$ ,  $\mathbf{R}_{12}$ , and  $\mathbf{R}_{22}$ , as per equation (20). Following step 120, step 130 computes  $\mathbf{R}_{ee,min}$  from equation (22) and adjusts  $\Delta$  to minimize the *trace* (or determinant) of  $\mathbf{R}_{ee,min}$ , computes  $\tilde{\mathbf{B}}_{opt}$  from equation (21), and from  $\tilde{\mathbf{B}}_{opt}$  determines the coefficients of the  $n_i \times n_i$  filters of element 26, pursuant to equation (10). Step 140 computes  $\mathbf{W}_{opt}^*$  from equation (14), and finally, step 150 installs the coefficients developed in step 130 into the filters of element 26 and the coefficients developed in step 140 into the filters of element 23.

A second approach for computing  $\tilde{\mathbf{B}}_{opt}$  utilizes the *block* Cholesky factorization (which is a technique that is well known in the art):

$$\begin{aligned} \mathbf{R} &\equiv \mathbf{R}_{xx}^{-1} + \mathbf{H}^* \mathbf{R}_{nn}^{-1} \mathbf{H} \\ &= \begin{bmatrix} \mathbf{L}_1 & \mathbf{0} \\ \mathbf{L}_2 & \mathbf{L}_3 \end{bmatrix} \begin{bmatrix} \mathbf{D}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_2 \end{bmatrix} \begin{bmatrix} \mathbf{L}_1^* & \mathbf{L}_2^* \\ \mathbf{0} & \mathbf{L}_3^* \end{bmatrix} \\ &\equiv \mathbf{LDL}^*, \end{aligned} \quad (22)$$

where  $\mathbf{L}_1$  is of size  $n_i(\Delta + 1) \times n_i(\Delta + 1)$ . Using the result in equations (18) and (19) yields

$$\begin{aligned} \tilde{\mathbf{B}}_{opt} &= \begin{bmatrix} \mathbf{I}_{n_i(\Delta+1)} \\ \mathbf{L}_2 \mathbf{L}_1^{-1} \end{bmatrix} \mathbf{C} = \begin{bmatrix} \mathbf{I}_{n_i} \\ \mathbf{L}_2 \mathbf{L}_1^{-1} \mathbf{C} \end{bmatrix} \\ &= \mathbf{L} \begin{bmatrix} \mathbf{e}_{n_i \Delta_{opt}} & \cdots & \mathbf{e}_{n_i(\Delta_{opt}+1)-1} \end{bmatrix} \end{aligned} \quad (23)$$

and

$$\begin{aligned}\mathbf{R}_{ee,min} &= \mathbf{C}^* \mathbf{D}_1^{-1} \mathbf{C} \\ &= \text{diag}(d_{n_i \Delta_{opt}}^{-1}, \dots, d_{n_i (\Delta_{opt}+1)-1}^{-1}),\end{aligned}\quad (24)$$

where the index  $\Delta_{opt}$  is chosen (as before) to minimize the trace and determinant of  $\mathbf{R}_{ee,min}$ . Using equation (23), equation (14) can be expressed as follows

$$\begin{aligned}\mathbf{W}_{opt}^* &= \tilde{\mathbf{B}}_{opt}^* \mathbf{R}_{xx} \mathbf{H}^* (\mathbf{H} \mathbf{R}_{xx} \mathbf{H}^* + \mathbf{R}_{nn})^{-1} \\ &= \tilde{\mathbf{B}}_{opt}^* (\mathbf{R}_{xx}^{-1} + \mathbf{H}^* \mathbf{R}_{nn}^{-1} \mathbf{H})^{-1} \mathbf{H}^* \mathbf{R}_{nn}^{-1} \\ &= \begin{bmatrix} d_{n_i \Delta_{opt}}^{-1} \mathbf{e}_{n_i \Delta_{opt}}^* \\ \vdots \\ d_{n_i (\Delta_{opt}+1)-1}^{-1} \mathbf{e}_{n_i (\Delta_{opt}+1)-1}^* \end{bmatrix} \mathbf{L}^{-1} \mathbf{H}^* \mathbf{R}_{nn}^{-1}\end{aligned}\quad (25)$$

5 Yet a third approach for computing  $\tilde{\mathbf{B}}_{opt}$  and  $\mathbf{R}_{ee,min}$  defines  $\tilde{\mathbf{B}}^* = [\mathbf{C}^* \quad \bar{\mathbf{B}}^*]$

and partitions  $\mathbf{R}^\perp$  into as  $\begin{bmatrix} \mathbf{R}_{11}^\perp & \mathbf{R}_{12}^\perp \\ \mathbf{R}_{12}^{*\perp} & \mathbf{R}_{22}^\perp \end{bmatrix}$ , where  $\mathbf{R}_{11}^\perp$  is of size  $n_i(\Delta+1) \times n_i(\Delta+1)$ , to

yield

$$\begin{aligned}\mathbf{R}_{ee} &= \tilde{\mathbf{B}}^* \mathbf{R}^\perp \tilde{\mathbf{B}} \\ &\equiv [\mathbf{C}^* \quad \bar{\mathbf{B}}^*] \begin{bmatrix} \mathbf{R}_{11}^\perp & \mathbf{R}_{12}^\perp \\ \mathbf{R}_{12}^{*\perp} & \mathbf{R}_{22}^\perp \end{bmatrix} \begin{bmatrix} \mathbf{C} \\ \bar{\mathbf{B}} \end{bmatrix} \\ &\equiv [\mathbf{I}_{n_i} \quad \bar{\mathbf{B}}^*] \begin{bmatrix} \bar{\mathbf{R}}_{11}^\perp & \bar{\mathbf{R}}_{12}^\perp \\ \bar{\mathbf{R}}_{12}^{*\perp} & \bar{\mathbf{R}}_{22}^\perp \end{bmatrix} \begin{bmatrix} \mathbf{I}_{n_i} \\ \bar{\mathbf{B}} \end{bmatrix} \\ &= (\bar{\mathbf{R}}_{11}^\perp - \bar{\mathbf{R}}_{12}^\perp (\mathbf{R}_{22}^\perp)^{-1} \bar{\mathbf{R}}_{12}^{*\perp}) + (\bar{\mathbf{B}}^* + \bar{\mathbf{R}}_{12}^\perp (\mathbf{R}_{22}^\perp)^{-1}) \mathbf{R}_{22}^\perp (\bar{\mathbf{B}}^* + \bar{\mathbf{R}}_{12}^\perp (\mathbf{R}_{22}^\perp)^{-1})^*,\end{aligned}\quad (26)$$

where  $\bar{\mathbf{R}}_{11}^\perp \equiv \mathbf{C}^* \mathbf{R}_{11}^\perp \mathbf{C}$  and  $\bar{\mathbf{R}}_{12}^\perp = \mathbf{C}^* \mathbf{R}_{12}^\perp$ . Therefore,

$$\mathbf{B}_{opt} = -\bar{\mathbf{R}}_{12}^\perp (\mathbf{R}_{22}^\perp)^{-1}\quad (27)$$

$$\mathbf{W}_{opt}^* = [\mathbf{0}_{n_i \times n_i \Delta} \quad \mathbf{I}_{n_i} - \bar{\mathbf{R}}_{12}^\perp (\mathbf{R}_{22}^\perp)^{-1}] (\mathbf{R}_{xx} \mathbf{H}^* (\mathbf{H} \mathbf{R}_{xx} \mathbf{H}^* + \mathbf{R}_{nn})^{-1})\quad (28)$$

$$\mathbf{R}_{ee,min} = \bar{\mathbf{R}}_{11}^\perp - \bar{\mathbf{R}}_{12}^\perp (\mathbf{R}_{22}^\perp)^{-1} \bar{\mathbf{R}}_{12}^{*\perp}\quad (28)$$

## Scenario 2

In this scenario it is assumed that users whose signals are received by the FIG. 1 receiver are ordered so that lower-indexed users are detected first, and current decisions from lower-indexed users are used by higher-indexed users in making their decisions, i.e.,  $\mathbf{B}_0$  is a lower-triangular matrix. The general results of

equations (21) and (22) can be applied by setting  $\mathbf{C}^* = \begin{bmatrix} \mathbf{0}_{n_i \times n_i \Delta} & \mathbf{B}_0^* \end{bmatrix}$  where  $\mathbf{B}_0$  is an  $n_i \times n_i$  monic lower-triangular matrix whose entries are optimized to minimize  $\text{trace}(\mathbf{R}_{ee, \min})$ . To this end, a partitioning can be defined where

$$\mathbf{R}_{11}^{-1} \equiv \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2 \\ \mathbf{R}_2^* & \mathbf{R}_3 \end{bmatrix}, \quad (29)$$

- 5  $\mathbf{R}_{11}$  being the term corresponding to  $\mathbf{R}_{11}$  of equation (20), with  $\mathbf{R}_1$  being of size  $n_i \Delta \times n_i \Delta$ , and  $\mathbf{R}_3$  being of size  $n_i \times n_i$ . Equation (22) simplifies to

$$\mathbf{R}_{ee, \min} = \mathbf{B}_0^* \mathbf{R}_3 \mathbf{B}_0 \quad (30)$$

It can be shown that, the optimum monic lower-triangular  $\mathbf{B}_0$  that minimizes  $\text{trace}(\mathbf{R}_{ee, \min})$  is given by the nomic lower-triangular Cholesky factor of  $\mathbf{R}_3^{-1}$ , i.e.,

$$\mathbf{R}_3^{-1} = \mathbf{L}_3 \mathbf{D}_3 \mathbf{L}_3^*, \quad (31)$$

which yields

$$\mathbf{B}_0^{\text{opt}} = \mathbf{L}_3, \quad (32)$$

and

$$\mathbf{R}_{ee, \min} = \mathbf{D}_3^{-1}. \quad (33)$$

- 15 The result is that  $\tilde{\mathbf{B}}_{\text{opt}} = \begin{bmatrix} \mathbf{I}_{n_i(\Delta+1)} \\ \mathbf{R}_{12}^* \mathbf{R}_{11}^{-1} \end{bmatrix} \mathbf{C}$ , as expressed in equation (21), with the modified value of  $\mathbf{R}_{11}^{-1}$ , and with

$$\mathbf{C}^* = \begin{bmatrix} \mathbf{0}_{n_i \times n_i \Delta} & \mathbf{B}_0^* \end{bmatrix}. \quad (34)$$

A second approach for computing the optimum FIR filter coefficients for the FIG. 1 receiver involves computing a standard – rather than a block – Cholesky factorization of the matrix  $\mathbf{R} = \mathbf{R}_{xx}^{-1} + \mathbf{H}^* \mathbf{R}_{nn}^{-1} \mathbf{H}$  (see the definition following equation (15)) in the form  $\mathbf{L} \mathbf{D} \mathbf{L}^*$ . Then, the coefficients of the element 23 filters is given by the  $n_i$  adjacent columns of  $\mathbf{L}$  that correspond to a diagonal matrix with the smallest trace. Therefore, equations (23) and (25) are used to compute the corresponding coefficients, with the understanding that  $\mathbf{L}$  is now a lower-triangular matrix, rather than a *block* lower-triangular matrix. The equivalence of the two approaches can be shown using the nesting property of Cholesky factorization.

FIG. 4 presents a flowchart for carrying out the method of determining the filter coefficients that processor 22 computes pursuant to scenario 2. Steps 100 through 120 are the same as in FIG. 3, but the method diverges somewhat in the following steps. In step 131 the partition according to equation (20) is developed for a  $\Delta$  that minimizes  $\mathbf{R}_{ee,min}$  of equation (33), and control passes to step 141, where  $\mathbf{B}_0^{opt}$  is computed based on equations (31) and (32), followed by a computation of  $\tilde{\mathbf{B}}_{opt}$  based on equations (21) and (34). Following step 141, step 151 computes  $\mathbf{W}_{opt}^*$  from equation (14), and finally, step 161 installs the coefficients developed in step 141 into the filters of element 26 and the coefficients developed in step 151 into the filters of element 23.

### Scenario 3

When multistage detectors are employed, current decisions from all other users, obtained from a previous detection stage, are available to the user of interest. Therefore, suppressing their interfering effects would improve the performance of the receiver. This detection scenario has the same mathematical formulation as scenarios 1 and 2, except that  $\mathbf{B}_0$  is now constrained only to be monic, i.e.,  $\mathbf{e}_i^* \mathbf{B}_0 \mathbf{e}_i = 1$  for all  $0 \leq i \leq n_i - 1$ . The general results in equations (21) and (22) still apply with  $\mathbf{C}^* = \begin{bmatrix} \mathbf{0}_{n_i \times n_i \Delta} & \mathbf{B}_0^* \end{bmatrix}$  where  $\mathbf{B}_0$  is optimized to minimize  $trace(\mathbf{R}_{ee,min})$ . In short, under scenario 3, the following optimization problem is solved:

$$\min_{\mathbf{B}_0} trace(\mathbf{B}_0^* \mathbf{R}_3 \mathbf{B}_0) \text{ subject to } \mathbf{e}_i^* \mathbf{B}_0 \mathbf{e}_i = 1 \text{ for all } 0 \leq i \leq n_i - 1, \quad (33)$$

where  $\mathbf{R}_3$  is as defined in equation (29). Using Lagrange multiplier techniques, it can be shown that the optimum monic  $\mathbf{B}_0$  and the corresponding MMSE are given by

$$\mathbf{B}_0^{opt} = \frac{\mathbf{R}_3^{-1} \mathbf{e}_{i-1}}{\mathbf{e}_i^* \mathbf{R}_3^{-1} \mathbf{e}_i}; \quad 0 \leq i \leq n_i - 1. \quad (34)$$

Thus, the method of determining the filter coefficients that processor 22 computes pursuant to scenario 3 is the same as the method depicted in FIG. 4, except that the computation of  $\mathbf{B}_0^{opt}$  within step 141 follows the dictates of equation (34).

With the above analysis in mind, a design of the filter coefficients of the filters within elements 23 and 26 can proceed for any given set of system parameters, which includes:

- MIMO channel memory between the input points and the output point of the actual transmission channel,  $\nu$ ,
- The number of pre-filter taps chosen,  $N_f$ ,
- The shortened MIMO memory,  $N_b$ ,
- The number of inputs to the transmission channel,  $n_i$ ,
- The number of output derived from the transmission channel,  $n_o$ ,
- The autocorrelation matrix of the inputs,  $\mathbf{R}_{xx}$ ,
- The autocorrelation matrix of the noise,  $\mathbf{R}_{nn}$ ,
- The oversampling used,  $l$ , and
- The decision delay,  $\Delta$ .

It should be understood that a number of aspects of the above disclosure are merely illustrative, and that persons skilled in the art may make various modifications that, nevertheless, are within the spirit and scope of this invention.



**Claims:**

1. A receiver responsive to an  $n_o$  plurality of entry points comprising:  
a feedforward filter structure having an  $n_o \times n_i$  plurality of FIR filters, each  
responsive to a signal that is derived from one of said  $n_o$  entry points and each  
developing an output signal that contributes to one of  $n_i$  feedforward filter structure  
outputs;  
a feedback filter structure developing  $n_i$  feedback signals, the structure  
having an  $n_i \times n_i$  plurality of FIR filters, each being responsive to one of  $n_i$  output  
signals;  
a subtractor structure that develops  $n_i$  signals from signals of said  $n_i$   
feedforward filter structure outputs and said  $n_i$  feedback signals; and  
decision logic responsive to said  $n_i$  outputs developed by said subtractor  
structure, for developing said  $n_i$  output signals.

2. The receiver of claim 1 further comprising a sampling circuit interposed  
between said  $n_o$  plurality of entry points and said feedforward filter structure that  
samples received signal at rate  $T_s = \frac{T}{l}$ , where  $l$  is an integer and  $T$  is symbol rate  
of a transmitter whose signals said receiver receives.

3. The receiver of claim 2 where  $l > 1$ .

4. The receiver of claim 1 where further including a processor coupled to  
signals applied to said feedforward filter structure, for computing coefficients of  
said FIR filters included in said feedforward filter structure and of said FIR filters  
included in said feedback filter structure.

5. The receiver of claim 4 where coefficients are computed in said  
processor in response to a block of  $N_f$  symbols.

6. The receiver of claim 4 where said coefficients are computed in a non-iterative manner.

7. The receiver of claim 4 where said coefficients are computed with a non-iterative equation.

8. The receiver of claim 4 where said coefficients are computed once every time interval that is related to rapidity of change in characteristics of transmission medium preceding said entry points.

9. The receiver of claim 8 where said processor installs computed coefficients of said FIR filters in said FIR filters following each computation.

10. The receiver of claim 1 where said FIR filters in said feedforward filter structure form an array of filters that includes one FIR filter connected between each of said  $n_o$  input points and said  $n_i$  outputs.

11. The receiver of claim 10 where said entry points are antennas and said  $n_o$  plurality of antennas receive signals, via a transmission channel, from a transmitter having a multiple number of transmitting antennas.

12. The receiver of claim 11 where said transmitter has  $n_i$  transmitting antennas.

13. The receiver of claim 2 where said plurality of FIR filters in said feedforward structure is expressed by matrix  $\mathbf{W}$ , and  $\mathbf{W}$  is computed

by  $\mathbf{W}_{opt}^* = \tilde{\mathbf{B}}_{opt}^* \mathbf{R}_{xy} \mathbf{R}_{yy}^{-1}$ ,  $\mathbf{W}_{opt}^* = \tilde{\mathbf{B}}_{opt}^* \mathbf{R}_{xx} \mathbf{H}^* (\mathbf{H} \mathbf{R}_{xx} \mathbf{H}^* + \mathbf{R}_{nn})^{-1}$ , or

$\mathbf{W}_{opt}^* = \tilde{\mathbf{B}}_{opt}^* (\mathbf{R}_{xx}^{-1} + \mathbf{H}^* \mathbf{R}_{nn}^{-1} \mathbf{H})^{-1} \mathbf{H}^* \mathbf{R}_{nn}^{-1}$ , where  $\mathbf{R}_{xx}$  is an autocorrelation matrix of a block of signals transmitted by a plurality of transmitting antennas to said  $n_o$  antennas via a channel having a transfer characteristic  $\mathbf{H}$ ,  $\mathbf{R}_{nn}$  is an autocorrelation matrix of

noise received by said plurality of  $n_o$  antennas during said block of signals transmitted by said transmitting antennas,  $\mathbf{R}_{xy} = \mathbf{R}_{xx}\mathbf{H}^*$ ,  $\mathbf{R}_{yy} = \mathbf{H}\mathbf{R}_{xx}\mathbf{H}^* + \mathbf{R}_{nn}$ , and  $\tilde{\mathbf{B}}_{opt}^*$  is a sub-matrix of matrix  $\mathbf{B}_{opt}^*$ , where  $\mathbf{B}_{opt} = \text{argmin}_B \text{trace}(\mathbf{R}_{ee})$  subject to a selected constraint,  $\mathbf{R}_{ee}$  being the error autocorrelation function.

5

**14.** The receiver of claim **13** where said plurality of FIR filters in said feedback structure is expressed by matrix  $\begin{bmatrix} \mathbf{I}_{n_i} & \mathbf{0}_{n_i \times n_i N_b} \end{bmatrix} - \mathbf{B}^*$ .

**15.** The receiver of claim **1** where coefficients of the FIR filters in said feedforward filter are set to results in an effective transmission channel  $\mathbf{B}$  with memory  $N_b$ , where  $N_b \leq \nu$ , where  $\mathbf{B}$  is optimized so that  $\mathbf{B}_{opt} = \text{argmin}_B \text{trace}(\mathbf{R}_{ee})$  subject to a selected constraint;  $\mathbf{R}_{ee}$  being the error autocorrelation function, the feedback filter is modeled by  $\begin{bmatrix} \mathbf{I}_{n_i} & \mathbf{0}_{n_i \times n_i N_b} \end{bmatrix} - \mathbf{B}^*$ , where  $n_i$  is the number of outputs in the feedforward filter, as well as the number of outputs in the feedback filter, and the feedforward filter is modeled by  $\mathbf{W}$ , where  $\mathbf{W}_{opt}^* = \tilde{\mathbf{B}}_{opt}^* \mathbf{R}_{xy} \mathbf{R}_{yy}^{-1}$ ,  $\mathbf{R}_{xy}$  is the cross correlation between transmitted signals and signals received by said receiver, and  $\mathbf{R}_{yy}$  is the autocorrelation of the received signals.

**16.** The receiver of claim **15** where said selected constraint is  $\tilde{\mathbf{B}}^* \Phi = \mathbf{C}^*$ ,

20 where  $\Phi \equiv \begin{bmatrix} \mathbf{I}_{n_i} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{n_i} & \vdots & \vdots \\ \vdots & \mathbf{0} & \ddots & \mathbf{I}_{n_i} \\ \mathbf{0} & \dots & \dots & \mathbf{0} \end{bmatrix}$  and  $\mathbf{C}^* = \begin{bmatrix} \mathbf{0}_{n_i \times n_i \Delta} & \mathbf{I}_{n_i} \end{bmatrix}$ .

17. The receiver of claim 15 where said selected constraint is  $\tilde{\mathbf{B}}^* \Phi = \mathbf{C}^*$ ,

where  $\Phi \equiv \begin{bmatrix} \mathbf{I}_{n_i} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{n_i} & \vdots & \vdots \\ \vdots & \mathbf{0} & \ddots & \mathbf{I}_{n_i} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} \end{bmatrix}$  and  $\mathbf{C}^* = \begin{bmatrix} \mathbf{0}_{n_i \times n_i \Delta} & \mathbf{B}_0^* \end{bmatrix}$ ,  $\mathbf{B}_0^*$  being a monic lower-

triangular matrix whose entries are optimized to minimize  $\text{trace}(\mathbf{R}_{ee,min})$

5 18. The receiver of claim 15 where said selected constraint is  $\mathbf{e}_i^* \mathbf{B}_0 \mathbf{e}_i = 1$ , where  $\mathbf{e}_i$  is a vector with value 1 in position  $i$  and values 0 elsewhere, and where  $\mathbf{B}_0^*$  being a monic lower-triangular matrix whose entries are optimized to minimize  $\text{trace}(\mathbf{R}_{ee,min})$ .

10 19. The receiver of claim 13 wherein said plurality of FIR filters in said feedback filter structure and in said feedforward filter structure are subjected to designer constraints relative to any one or a number of members of the following set: transmission channel memory, size of said block, effective memory of the combination consisting of said transmission channel;  $n_i$ ,  $n_o$ , autocorrelation matrix  $\mathbf{R}_{xx}$ , autocorrelation matrix  $\mathbf{R}_{nn}$ , value of factor  $l$  in said sampling circuit, and  
15 decision delay.

20. The receiver where said matrix  $\mathbf{W}$  is expressible by

$\mathbf{W} = [\mathbf{W}_0 \quad \mathbf{W}_1 \quad \cdots \quad \mathbf{W}_{N_f-1}]^t$ , where matrix  $\mathbf{W}_q$  is a matrix that specifies  $q^{\text{th}}$  tap

coefficients of said FIR filters.

## 5

A MIMO Decision Feedback Equalizer improves operation of a receiver by cancelling the spatio-temporal interference effects caused by the MIMO channel memory with a set of FIR filters in both the feedforward and the feedback MIMO filters. The coefficients of these FIR filters can be fashioned to provide a variety of controls by the designer.

FIG. 1

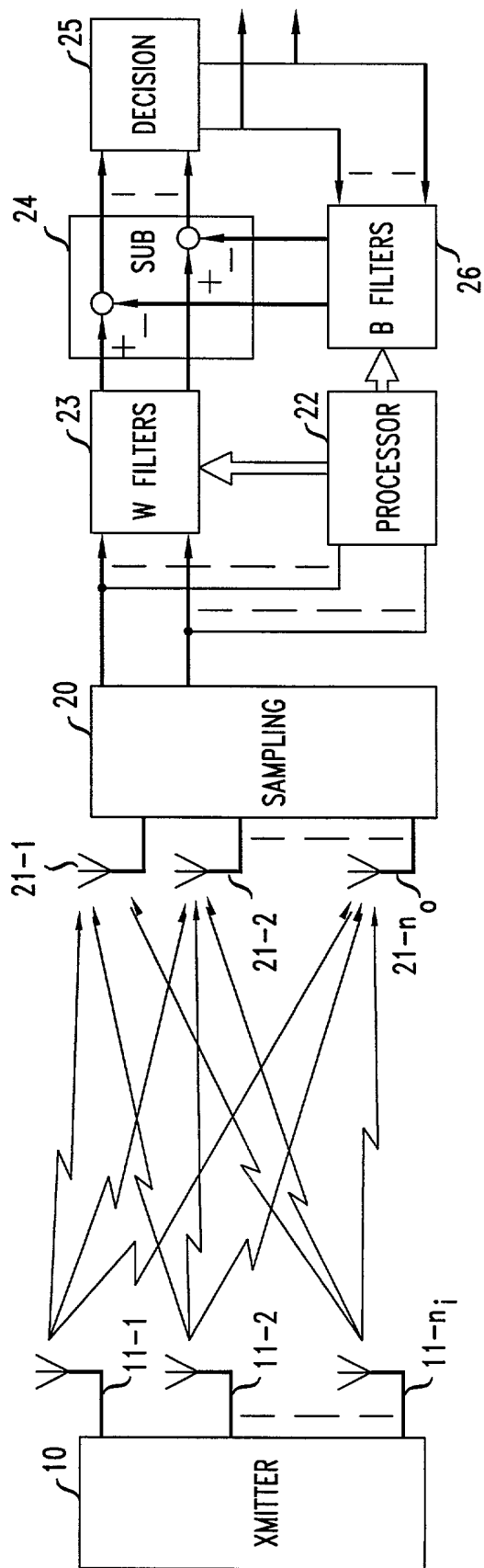


FIG. 2

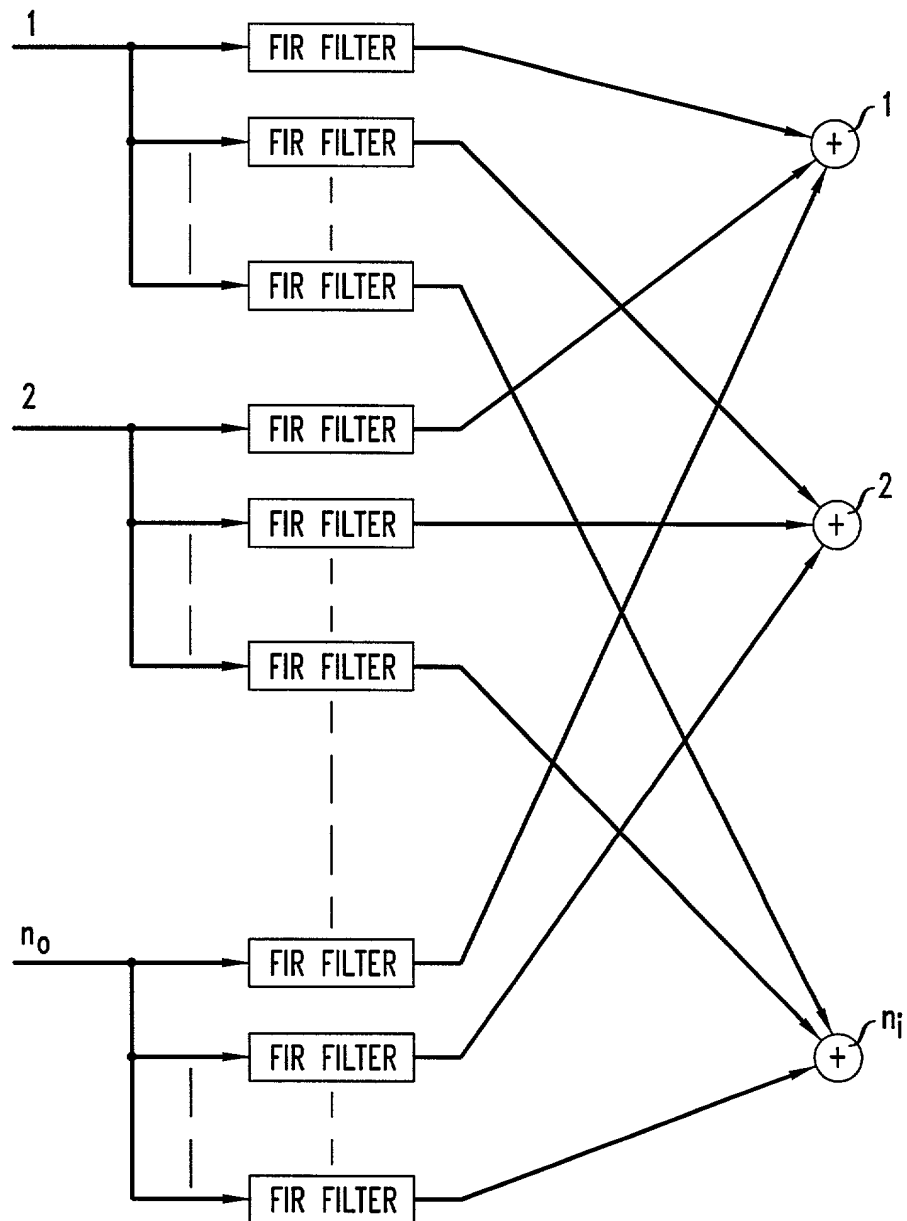


FIG. 3

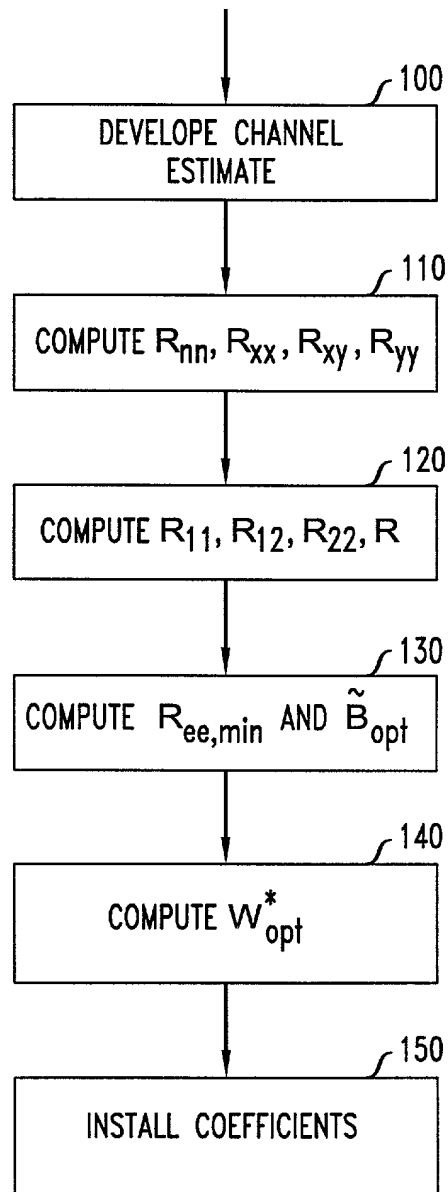
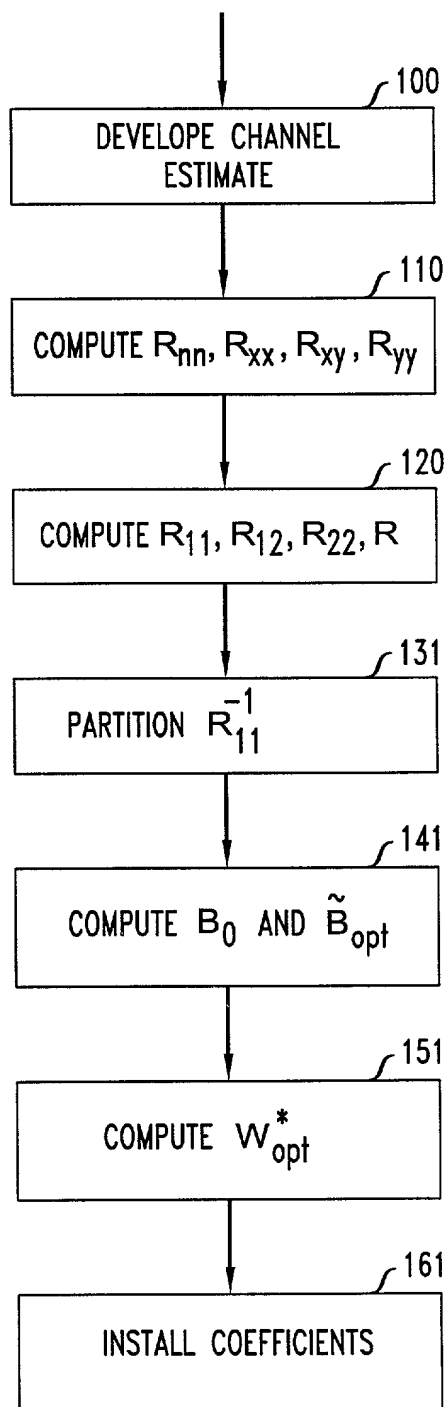




FIG. 4



IN THE UNITED STATES  
PATENT AND TRADEMARK OFFICE

**Declaration and Power of Attorney**

As a below named inventor, I hereby declare that:

My residence, post office address and citizenship are as stated below next to my name.

I believe I am an original, first and sole inventor of the subject matter which is claimed and for which a patent is sought on the invention entitled **Finite-Length Equalization Over Multi-Input Multi-Output Channels** the specification of which was filed on 10/8/99, as provisional application Serial No. 60/158714.

I hereby state that I have reviewed and understand the contents of the above-identified specification, including the claims, as amended by an amendment, if any, specifically referred to in this oath or declaration.

I acknowledge the duty to disclose all information known to me, which is material to patentability as defined in Title 37, Code of Federal Regulations, 1.56.

I hereby claim foreign priority benefits under Title 35, United States Code, 119 of any foreign application(s) for patent or inventors' certificate listed below and have also identified below any foreign application for patent or inventors' certificate having a filing date before that of the application on which priority is claimed:

I hereby claim the benefit under Title 35, United States Code, 120 of any United States application(s) listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of Title 35, United States Code, 112, we acknowledge the duty to disclose all information known to us to be material to patentability as defined in Title 37, Code of Federal Regulations, 1.56 which became available between the filing date of the prior application and the national or PCT international filing date of this application:

Provisional Application No. 60/158,714, filed October 8, 1999

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

008260-88972960

I hereby appoint the following attorney(s) with full power of substitution and revocation, to prosecute said application, to make alterations and amendments therein, to receive the patent, and to transact all business in the Patent and Trademark Office connected therewith:

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I also appoint the following as associate attorney(s), with full power to prosecute said application, to make alternations and amendments therein, and to transact all business in the Patent and Trademark Office connected therewith:

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